On automorphism groups of semigroups of k-linked upfamilies

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The through study of various extensions of semigroups was started in [12] and continued in [1]-[10], [13]-[19]. The largest among these extensions is the semigroup $\upsilon(S)$ of all upfamilies on a semigroup S. A family $\mathcal{M}$ of non-empty subsets of a set X is called *an upfamily* if for each set $A \in \mathcal{M}$ any subset $B \supset A$ of X belongs to $\mathcal{M}$. Each family $\mathcal{B}$ of non-empty subsets of X generates the upfamily $\{A \subset X : \exists B \in \mathcal{B}\ (B \subset A)\}$ which we denote by $\langle B \subset X : B \in \mathcal{B}\rangle$. An upfamily $\mathcal{F}$ that is closed under taking finite intersections is called a *filter*. A filter $\mathcal{U}$ is called an *ultrafilter* if $\mathcal{U} = \mathcal{F}$ for any filter $\mathcal{F}$ containing $\mathcal{U}$. The family $\beta(X)$ of all ultrafilters on a set X is called the *Stone-Čech compactification* of X, see [20]. An ultrafilter $\langle\{x\}\rangle$, generated by a singleton $\{x\}$, $x \in X$, is called *principal*. Each point $x \in X$ is identified with the principal ultrafilter $\langle\{x\}\rangle$ generated by the singleton $\{x\}$, and hence we can consider $X \subset \beta(X) \subset \upsilon(X)$. It was shown in [12] that any associative binary operation $* : S \times S \to S$ can be extended to an associative binary operation $* : \upsilon(S) \times \upsilon(S) \to \upsilon(S)$ by the formula

$$\mathcal{L} * \mathcal{M} = \Big\langle \bigcup_{a \in L} a * M_a : L \in \mathcal{L},\ \ \{M_a\}_{a \in L} \subset \mathcal{M} \Big\rangle$$

for upfamilies $\mathcal{L}, \mathcal{M} \in \upsilon(S)$. In this case the Stone-Čech compactification $\beta(S)$ is a subsemigroup of the semigroup $\upsilon(S)$. The semigroup $\upsilon(S)$ contains many other important extensions of S. In particular, it contains the semigroups $N_k(S)$ of k-linked upfamilies for $k \in \mathbb{N} \setminus \{1\}$. An upfamily $\mathcal{L} \in \upsilon(S)$ is called *k-linked* if $\bigcap \mathcal{F} \neq \emptyset$ for any subfamily $\mathcal{F} \subset \mathcal{L}$ of cardinality $|\mathcal{F}| \le k$.

Given a semigroup S we shall discuss the algebraic structure of automorphism groups $\mathrm{Aut}\,(N_k(S))$ of extensions $N_k(S)$ of S. We show that for each $k \in \mathbb{N}\setminus\{1\}$ any automorphism of a semigroup S can be extended to an automorphism of its extension $N_k(S)$, and the automorphism group $\mathrm{Aut}\,(N_k(S))$ of the extension $N_k(S)$ of a semigroup S contains a subgroup, isomorphic to the group $\mathrm{Aut}\,(S)$. We describe automorphism groups of extensions of groups, finite monogenic semigroups, null semigroups, almost null semigroups, right zero semigroups and left zero semigroups.

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