ON PELL–PADOVAN SEQUENCE

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The *Pell–Padovan sequence* $\{P_n\}_{n\geq 0}$ is the sequence of integers defined by a third-order recurrence equation

$$P_n = 2P_{n-2} + P_{n-3},$$

for $n \geq 3$, where $P_0 = P_1 = P_2 = 1$. The first few values of Pell–Padovan numbers are $1, 1, 1, 3, 3, 7, 9, 17, 25, 43, 67, 111, 177, 289, \ldots$ (see, for example, [1]).

We investigate some families of Toeplitz–Hessenberg determinants the entries of which are Pell–Padovan numbers. As a result, we obtain new identities involving sums of products of these numbers and multinomial coefficients. In particular we obtain some connection formulas between Pell–Padovan and Fibonacci numbers.

Our approach is similar in spirit to [2–4].

Recall that the Fibonacci sequence $\{F_n\}_{n\geq 0}$ is defined by the initial values $F_0 = 0$, $F_1 = 1$ and the recurrence relation

$$F_n = F_{n-1} + F_{n-2}, \quad n \geq 2.$$

Theorem. *Let $n \geq 2$, except when noted otherwise. Then*

$$\sum_{\sigma_n=n} (-1)^{|s|} m_n(s) P_1^{s_1} P_2^{s_2} \cdots P_n^{s_n} = 2(-1)^n F_{n-2},$$

$$\sum_{\sigma_n=n} (-1)^{|s|} m_n(s) P_2^{s_1} P_3^{s_2} \cdots P_{n+1}^{s_n} = (-1)^{\lfloor \frac{4n+1}{3} \rfloor} + (-1)^{\lfloor \frac{4n+2}{3} \rfloor},$$

$$\sum_{\sigma_n=n} (-1)^{|s|} m_n(s) P_3^{s_1} P_4^{s_2} \cdots P_{n+2}^{s_n} = 2(-1)^n F_{2n},$$

$$\sum_{\sigma_n=n} (-1)^{|s|} m_n(s) P_3^{s_1} P_5^{s_2} \cdots P_{2n+1}^{s_n} = -2F_{n-4}, \qquad n \geq 4,$$

$$\sum_{\sigma_n=n} (-1)^{|s|} m_n(s) P_0^{s_1} P_2^{s_2} \cdots P_{2n-2}^{s_n} = 2F_{n-1} - 2^{n-1}, \qquad n \geq 1,$$

$$\sum_{\sigma_n=n} (-1)^{|s|} m_n(s) P_2^{s_1} P_4^{s_2} \cdots P_{2n}^{s_n} = 2 \sum_{j=0}^{\lfloor (n-1)/3 \rfloor} (-1)^{j+1} \binom{n-1}{3j+1},$$

$$\sum_{\sigma_n=n} (-1)^{|s|} m_n(s) P_4^{s_1} P_6^{s_2} \cdots P_{2n+2}^{s_n} = (-1)^{\lfloor \frac{5n-2}{3} \rfloor} + (-1)^{\lfloor \frac{5n-1}{3} \rfloor},$$

where $\lfloor \cdot \rfloor$ is the floor function, $\sigma_n = s_1 + 2s_2 + \cdots + ns_n$, $|s| = s_1 + s_2 + \cdots + s_n$, $m_n(s) = \frac{(s_1+\cdots+s_n)!}{s_1!\cdots s_n!}$ is the multinomial coefficient, and the summation is over integers $s_i \geq 0$ satisfying $s_1 + 2s_2 + \cdots + ns_n = n$.

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