

B-1192 Proposed by T. Goy, Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine.

Let M_n be an $n \times n$ matrix given for all $n \geq 1$ by

$$M_n = \begin{pmatrix} F_1 & 1 & 0 & \dots & 0 & 0 & 0 \\ F_2 & F_1 & 1 & \dots & 0 & 0 & 0 \\ F_3 & F_2 & F_1 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \ddots & \dots & \dots & \dots \\ F_{n-1} & F_{n-2} & F_{n-3} & \dots & F_2 & F_1 & 1 \\ F_n & F_{n-1} & F_{n-2} & \dots & F_3 & F_2 & F_1 \end{pmatrix}.$$

Prove that

$$\det(M_n) = \begin{cases} 0, & \text{if } n \text{ is even,} \\ 1, & \text{if } n \text{ is odd.} \end{cases}$$

B-1193 Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain.

If $F_1^2, F_2^2, \dots, F_n^2$ are the square of the first n Fibonacci numbers, then find real numbers a_1, a_2, \dots, a_n satisfying $a_k > F_k^2, 1 \leq k \leq n$, and

$$\frac{1}{F_n F_{n+1}} \sum_{k=1}^n a_k < \frac{\alpha^2}{\alpha - 1}.$$

B-1194 Proposed by D. M. Băținețu–Giurgiu, Matei Basarab National College, Bucharest, Romania and Neculai Stanciu, George Emil Palade School, Buzău, Romania.

Prove that

$$\begin{aligned} & \frac{L_1}{(L_1^2 + L_2^2 + 2)^{m+1}} + \frac{L_2}{(L_1^2 + L_2^2 + L_3^2 + 2)^{m+1}} + \dots + \frac{L_n}{(L_1^2 + L_2^2 + \dots + L_{n+1}^2 + 2)^{m+1}} \\ & \geq \frac{(L_{n+2} - 1)^{m+1}}{L_{n+2}^{m+1} (L_{n+2} - 3)^m} \end{aligned}$$

for any positive integers n and m .

B-1195 Proposed by Jeremiah Bartz, Francis Marion University, Florence, SC.

Let G_i denote the generalized Fibonacci sequence given by $G_1 = a, G_2 = b$, and $G_i = G_{i-1} + G_{i-2}$ for $i \geq 3$. Let $m \geq 1$ and $k \geq 0$. Prove that the area A of the polygon with $n \geq 3$ vertices

$$(G_m, G_{m+k}), (G_{m+2k}, G_{m+3k}), \dots, (G_{m+(2n-2)k}, G_{m+(2n-1)k})$$

is

$$\frac{|\mu| F_k (F_{2k(n-1)} - (n-1) F_{2k})}{2}$$

where $\mu = a^2 + ab - b^2$.