B-1208 Proposed by Ivan V. Fedak, Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine.

For every positive integer n, find all real solutions of the following linear system of equations:

$$\begin{array}{ccccccccccccc}
F_1x_1 & + & x_2 & & & & & & & & & = & F_3, \\
F_2x_1 & + & F_1x_2 & + & x_3 & & & & & & & = & F_4, \\
F_3x_1 & + & F_2x_2 & + & F_1x_3 & + & \cdots & & & & & = & F_5, \\
\vdots & & \vdots & & \vdots & & & \ddots & & & & & \vdots \\
F_{n-1}x_1 & + & F_{n-2}x_2 & + & F_{n-3}x_3 & + & \cdots & + & x_n & & & = & F_{n+1}, \\
F_nx_1 & + & F_{n-1}x_2 & + & F_{n-2}x_3 & + & \cdots & + & F_1x_n & + & x_{n+1} & = & F_{n+2}, \\
F_{n+1}x_1 & + & F_nx_2 & + & F_{n-1}x_3 & + & \cdots & + & F_2x_n & + & F_1x_{n+1} & = & F_{n+3}-1.
\end{array}$$

B-1209 Proposed by Hideyuki Ohtsuka, Saitama, Japan.

The Tribonacci numbers T_n satisfy $T_0 = 0$, $T_1 = T_2 = 1$, and

$$T_n = T_{n-1} + T_{n-2} + T_{n-3}, \quad \text{for } n \geq 3.$$

Prove that

$$\sum_{k=1}^{n} T_{2k}T_{2k-1} = \left(\sum_{k=1}^{n} T_{2k-1}\right)^2$$

for any integer $n \geq 1$.

B-1210 Proposed by Taras Goy, Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine.

Prove that

$$\sum_{t_1+2t_2+\cdots+nt_n=n} (-1)^{n-s} \frac{s!}{t_1!\,t_2!\,\cdots\,t_n!} F_1^{t_1}F_2^{t_2}\cdots F_n^{t_n} = \frac{1-(-1)^n}{2},$$

where $s = t_1 + t_2 + \cdots + t_n$.

SOLUTIONS

A Telescoping Lucas Sum

B-1186 Proposed by Hideyuki Ohtsuka, Saitama, Japan.
(Vol. 54.2, May 2016)

Prove that

$$\sum_{n=0}^{\infty} \frac{(-1)^n L_{2^n}}{L_{2^{n+1}}+1} = 0.$$

Solution by Ángel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.

Since $L_{2k} + 2(-1)^k = L_k^2$, for any even number m, we have $L_{2m} = L_m^2 - 2$. This applies to every denominator in the given expression for $n > 0$, and hence,

$$\frac{(-1)^n L_{2^n}}{L_{2^{n+1}}+1} = \frac{(-1)^n L_{2^n}}{L_{2^n}^2-1} = \frac{(-1)^n(L_{2^n}-1+1)}{(L_{2^n}-1)(L_{2^n}+1)} = \frac{(-1)^n}{L_{2^n}+1} + \frac{(-1)^n}{L_{2^{n+1}}+1}.$$