B-1228 **Proposed by Hideyuki Ohtsuka, Saitama, Japan.**

For any integer $n \geq 0$, find the closed form expressions for the sums

(i) $S_n = \sum_{i=0}^{n} \sum_{j=0}^{n} L_{3^i} L_{3^j} L_{2(3^i - 3^j)}$;

(ii) $T_n = \sum_{i=0}^{n} \sum_{j=0}^{n} F_{2\cdot 5^i} F_{2\cdot 5^j} L_{3(5^i - 5^j)}$.

B-1229 **Proposed by D. M. Bătineţu-Giurgiu, Matei Basarab National College, Bucharest, Romania, and Neculai Stanciu, George Emil Palade School, Bazău, Romania.**

Let $m, p \geq 0$. Evaluate

$$\lim_{n\to\infty} \left(\frac{\sqrt[n+1]{\big((2n+1)!!\big)^{m+1} F_{n+1}^{p(m+1)}}}{(n+1)^m} - \frac{\sqrt[n]{\big((2n-1)!!\big)^{m+1} F_{n}^{p(m+1)}}}{n^m} \right),$$

and

$$\lim_{n\to\infty} \left(\frac{\sqrt[n+1]{\big((2n+1)!!\big)^{m+1} L_{n+1}^{p(m+1)}}}{(n+1)^m} - \frac{\sqrt[n]{\big((2n-1)!!\big)^{m+1} L_{n}^{p(m+1)}}}{n^m} \right).$$

B-1230 **Proposed by T. Goy, Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine.**

For all integers $n \geq 0$, prove that

$$F_{2n+1} = (-1)^n \sum_{\substack{t_1, t_2, \ldots, t_n \geq 0 \\ t_1 + 2t_2 + \cdots + nt_n = n}} (-1)^{t_1 + t_3 + \cdots + t_{n-[1+(-1)^n]/2}} \frac{(t_1 + t_2 + \cdots + t_n)!}{t_1!\, t_2! \cdots t_n!} \cdot 2^{t_1}.$$

SOLUTIONS

Two Doses of AM-GM Inequality

B-1206 **Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain. (Vol. 55.2, May 2017)**

Let $n \geq 2$ be an integer. Prove that

$$1 + \frac{1}{n^2} \sum_{1 \leq i < j \leq n} \frac{\left(\sqrt{F_i F_{j+1}} - \sqrt{F_{i+1} F_j}\,\right)^2}{F_i F_j} \leq \frac{1}{n} \sum_{k=1}^{n} \frac{F_{k+1}}{F_k},$$

in which the subscripts are taken modulo n.

Solution by Ivan V. Fedak, Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine.

To begin, we rewrite the given inequality in the form