Taras Goy

On connection between some number sequences using Hessenberg matrices

Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine
E-mail: tarasgoy@yahoo.com

Let F_n, P_n, J_n and T_n be the Fibonacci, Pell, Jacobsthal and tribonacci numbers defined, for all integers $n \geq 0$, by $F_n = F_{n-1} + F_{n-2}$, $F_0 = 0$, $F_1 = 1$; $P_n = 2P_{n-1} + P_{n-2}$, $P_0 = 0$, $P_1 = 1$; $J_n = J_{n-1} + 2J_{n-2}$, $J_0 = 0$, $J_1 = 1$; $T_n = T_{n-1} + T_{n-2} + T_{n-3}$, $T_0 = T_1 = 0$, $T_2 = 1$, respectively.

Теорема 1 *Let $n \geq 1$, except when noted otherwise. Then*

$$\begin{aligned}
F_n &= \sum_{\sigma_n = n} (-1)^{|s|+1} p_n(s) P_1^{s_1} P_2^{s_2} \cdots P_n^{s_n}, \\
F_{n+1} &= 2^{1-n} \sum_{\sigma_n = n} p_n(s) J_2^{s_1} J_3^{s_2} \cdots J_{n+1}^{s_n}, \qquad n \geq 2, \\
F_{n-1} &= \sum_{\sigma_n = n} (-1)^{|s|+1} p_n(s) J_0^{s_1} J_1^{s_2} \cdots J_{n-1}^{s_n}, \\
F_{n-2} &= \sum_{\sigma_n = n} (-1)^{|s|+1} p_n(s) T_0^{s_1} T_1^{s_2} \cdots T_{n-1}^{s_n}, \\
F_{2n+3} &= \sum_{\sigma_n = n} (-1)^{|s|+n} p_n(s) P_3^{s_1} P_4^{s_2} \cdots P_{n+2}^{s_n}, \\
P_{n+2} &= \sum_{\sigma_n = n} (-1)^{|s|+1} p_n(s) T_2^{s_1} T_3^{s_2} \cdots T_{n+1}^{s_n}, \\
P_{n+2} &= \sum_{\sigma_n = n} (-1)^{n+|s|} p_n(s) F_5^{s_1} F_7^{s_2} \cdots F_{2n+3}^{s_n}, \qquad n \geq 2,
\end{aligned}$$

where $\sigma_n = s_1 + 2s_2 + \cdots + ns_n$, $|s| = s_1 + \cdots + s_n$ $(s_i \geq 0)$, and $p_n(s) = \binom{|s|}{s_1, \ldots, s_n}$ is the multinomial coefficients.

Our approach is similar in spirit to [1, 2].

[1] Goy T. *On new identities for Mersenne numbers* // Applied Mathematics E-Notes. – 2018. – **18**. – P. 100–105.

[2] Goy T. *Some families of identities for Padovan numbers* // Proceedings of the Jangjeon Mathematical Society. – 2018. – **21**, no. 3. – P. 413–419.