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Weakly M-preopen functions in biminimal structure spaces

Basumatary A.A.[1,2], Sarma D.J.[1], Tripathy B.C.[2]

The intention of this article is to define the concept of weakly M-preopen function in biminimal structure spaces. Several properties of this function have been established and its relationship with some other notions related to M-preopen sets in biminimal spaces have been investigated.

Key words and phrases: biminimal space, $M_{ij(X)}$-preopen set, $M_{ij(X)}$-preclosed set, $M_{ij(X)}$-θ-open set, $M_{ij(X)}$-θ-closed set, M_{ij}-weakly preopen function.

[1] Central Institute of Technology, Rangalikhata, Balagaon, Kokrajhar, Assam-783370, India
[2] Tripura University, Suryamaninagar, Agartala-799022, India
E-mail: albisbasu@gmail.com (Basumatary A.A.), djs_math@rediffmail.com (Sarma D.J.), tripathybc@rediffmail.com (Tripathy B.C.)

Introduction

Preopen sets play a very important role in the generalization of various types of continuous like functions in topological spaces. It was J.C. Kelly [6] who first introduced the concept of bitopological spaces. M. Jelic [4], A. Kar and P. Bhattacharyya [5], F.H. Khedr et al. [7] defined and studied the notion of preopen sets, precontinuous and preopen functions in bitopological spaces. T. Noiri and V. Popa [12] introduced weakly precontinuous functions in bitopological spaces which was followed by the same authors in 2006 (see [11]) with the concept of weakly open functions on the same spaces and established some of their properties. The study of minimal structure spaces was initiated by H. Maki et al. [8] in 1999. Then in 2000, V. Popa and T. Noiri [15] introduced and investigated M-continuous and other types of continuous functions on spaces with minimal structures. W.K. Min and Y.K. Kim [9] introduced m-preopen sets and m-precontinous functions on minimal spaces. It was T. Noiri [10] who introduced the notion of bi-m-spaces as a space with two minimal structures which were later in 2010 reintroduced as biminimal structure spaces by C. Boonpok [1]. C. Boonpok introduced in [2] the concept of m-preopen sets and studied the notion of M-continuous and weakly M-continuous functions in biminimal spaces. It is also found in the literature, that C. Carpintero et al. [3] had introduced and characterized the concepts of m-preopen sets and their related notions in biminimal spaces. In 2011, W. Phosri et al. [14] defined weakly M-precontinous functions on biminimal spaces and obtained several properties of these functions.

1 Preliminaries

In this section, we list some of those known definitions and results that will be used in preparing this article collected from different research papers. Throughout this paper,

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$(X, M_{1(X)}, M_{2(X)})$ (respectively, $(X, M_{(X)})$) denotes a biminimal space (respectively, minimal space) with minimal structures $M_{1(X)}$ and $M_{2(X)}$ (respectively, $M_{(X)}$) on a non-empty set X.

Definition 1 ([8]). *A collection $M_{(X)}$ of a powerset $P(X)$ of a non-empty set X is said to be a minimal structure on X if $\varnothing \in M_{(X)}$ and $X \in M_{(X)}$. By $(X, M_{(X)})$, we mean a minimal space. Members of $M_{(X)}$ are called $M_{(X)}$-open sets and the complement of $M_{(X)}$-open sets are called $M_{(X)}$-closed sets. That is, for a subset S of X, $S \in M_{(X)}$ means S is $M_{(X)}$-open and $X \setminus S \in M_{(X)}$ means S is $M_{(X)}$-closed.*

Definition 2 ([8]). *Let $(X, M_{(X)})$ be a minimal space and $S \subset X$. Then the $M_{(X)}$-closure of S and the $M_{(X)}$-interior of S, denoted by $M_{(X)}$-$Cl(S)$ and $M_{(X)}$-$Int(S)$, respectively, are defined as $M_{(X)}\text{-}Cl(S) = \bigcap \{T : S \subset T, X \setminus T \in M_{(X)}\}$ and $M_{(X)}\text{-}Int(S) = \bigcup \{T : T \subset S, T \in M_{(X)}\}$.*

Lemma 1 ([8]). *Let $(X, M_{(X)})$ be a minimal space and $S \subset X$, then*

(a) $M_{(X)}\text{-}Cl(X \setminus S) = X \setminus M_{(X)}\text{-}Int(S)$ and $M_{(X)}\text{-}Int(X \setminus S) = X \setminus M_{(X)}\text{-}Cl(S)$;

(b) $M_{(X)}\text{-}Int(S) \in M_{(X)}$ and $M_{(X)}\text{-}Cl(S)$ is $M_{(X)}$-closed;

(c) S is $M_{(X)}$-closed set if and only if $M_{(X)}\text{-}Cl(S) = S$ and $S \in M_{(X)}$ if and only if $M_{(X)}\text{-}Int(S) = S$;

(d) $S \subseteq M_{(X)}\text{-}Cl(S)$ and $M_{(X)}\text{-}Int(S) \subseteq S$;

(e) $M_{(X)}\text{-}Cl(M_{(X)}\text{-}Cl(S)) = M_{(X)}\text{-}Cl(S)$ and $M_{(X)}\text{-}Int(M_{(X)}\text{-}Int(S)) = M_{(X)}\text{-}Int(S)$.

Lemma 2 ([8]). *Let $(X, M_{(X)})$ be a minimal space and $S \subset X$. Then $a \in M_{(X)}\text{-}Cl(S)$ if and only if $T \cap S \neq \varnothing$ for every $T \in M_{(X)}$ containing a.*

Definition 3 ([1]). *A space $(X, M_{1(X)}, M_{2(X)})$ with two minimal structures $M_{1(X)}$ and $M_{2(X)}$ on a non-empty set X is called a biminimal structure space (briefly, biminimal space). If $S \subset X$, then the $M_{(X)}$-closure of S and the $M_{(X)}$-interior of S with respect to $M_{i(X)}$ are denoted by $M_{i(X)}\text{-}Cl(S)$ and $M_{i(X)}\text{-}Int(S)$, respectively, where $i = 1, 2$. If $S \in M_{i(X)}$, then we say that S is $M_{i(X)}$-open set and if $X \setminus S \in M_{i(X)}$ then S is $M_{i(X)}$-closed set.*

Definition 4 ([2]). *Let $(X, M_{1(X)}, M_{2(X)})$ be a biminimal space. Then a subset S of X is said to be*

(a) $M_{ij(X)}$-preopen if $S \subseteq M_{i(X)}\text{-}Int(M_{j(X)}\text{-}Cl(S))$ and $M_{ij(X)}$-preclosed if $X \setminus S$ is $M_{ij(X)}$-preopen;

(b) $M_{ij(X)}$-regular open if $S = M_{i(X)}\text{-}Int(M_{j(X)}\text{-}Cl(S)$;

(c) $M_{ij(X)}$-regular closed if $S = M_{i(X)}\text{-}Cl(M_{j(X)}\text{-}Int(S))$;

(d) $M_{ij(X)}$-α-open if $S \subseteq M_{i(X)}\text{-}Int(M_{j(X)}\text{-}Cl(M_{i(X)}\text{-}Int(S)))$, where $i, j = 1, 2$ and $i \neq j$.

Here we denote the family of all $M_{ij(X)}$-preopen, $M_{ij(X)}$-preclosed, $M_{ij(X)}$-regular open, $M_{ij(X)}$-regular closed and $M_{ij(X)}$-α-open sets by $M_{ij(X)}\text{-}PO(X)$, $M_{ij(X)}\text{-}PC(X)$, $M_{ij(X)}\text{-}RO(X)$, $M_{ij(X)}\text{-}RC(X)$ and $M_{ij(X)}\text{-}\alpha O(X)$, respectively.

Definition 5 ([3]). *Let $(X, M_{1(X)}, M_{2(X)})$ be a biminimal space and $S \subset X$. Then a point $a \in X$ is said to be*

(a) $M_{ij(X)}$-preinterior point of S if there exists $T \in M_{ij(X)}$-$PO(X)$ such that $a \in T \subset S$;

(b) $M_{ij(X)}$-precluster point of S if $T \cap S \neq \varnothing$ for every $T \in M_{ij(X)}$-$PO(X)$ containing a.

The set of all $M_{ij(X)}$-preinterior points of S is called $M_{ij(X)}$-preinterior of S and it is denoted by $M_{ij(X)}$-$Int_p(S)$. Also, the set of all $M_{ij(X)}$-precluster points of S is called $M_{ij(X)}$-preclosure of S and it is denoted by $M_{ij(X)}$-$Cl_p(S)$.

Lemma 3 ([3]). *Let $(X, M_{1(X)}, M_{2(X)})$ be a biminimal space and $S \subset X$. Then*

(a) $M_{ij(X)}$-$Int_p(S) \in M_{ij(X)}$-$PO(X)$;

(b) $M_{ij(X)}$-$Cl_p(S) \in M_{ij(X)}$-$PC(X)$;

(c) $M_{ij(X)}$-$Int_p(S) = \bigcup \{T : T \subset S$ and $T \in M_{ij(X)}$-$PO(X)\}$;

(d) $M_{ij(X)}$-$Cl_p(S) = \bigcap \{T : S \subset T$ and $T \in M_{ij(X)}$-$PC(X)\}$;

(e) $M_{ij(X)}$-$Int_p(S)$ is the largest $M_{ij(X)}$-preopen set in X contained in S;

(f) $M_{ij(X)}$-$Cl_p(S)$ is the smallest $M_{ij(X)}$-preclosed set in X containing S;

(g) $S \in M_{ij(X)}$-$PO(X)$ if and only if $S = M_{ij(X)}$-$Int_p(S)$ and $S \in M_{ij(X)}$-$PC(X)$ if and only if $S = M_{ij(X)}$-$Cl_p(S)$.

Definition 6 ([13]). *Let $(X, M_{1(X)}, M_{2(X)})$ be a biminimal space and $S \subset X$. A point $a \in X$ is said to be $M_{ij(X)}$-θ-adherent point of S if $S \cap M_{j(X)}$-$Cl(T) \neq \varnothing$ for all $T \in M_{i(X)}$ containing a. The set of all $M_{ij(X)}$-θ-adherent points of S is called $M_{ij(X)}$-θ-closure of S and it is denoted by $M_{ij(X)}$-$Cl_\theta(S)$. If $S = M_{ij(X)}$-$Cl_\theta(S)$, then S is said to be $M_{ij(X)}$-θ-closed. A subset S of X is $M_{ij(X)}$-θ-open if $X \setminus S$ is $M_{ij(X)}$-θ-closed. The $M_{ij(X)}$-θ-interior of S, denoted by $M_{ij(X)}$-$Int_\theta(S)$, is defined as the union of all $M_{ij(X)}$-θ-open sets contained in S. Therefore, $a \in M_{ij(X)}$-$Int_\theta(S)$ if and only if there exists $T \in M_{i(X)}$ containing a such that $a \in T \subset M_{j(X)}$-$Cl(T) \subset S$. We denote the family of all $M_{ij(X)}$-θ-closed sets and $M_{ij(X)}$-θ-open sets of X by $M_{ij(X)}$-$\theta C(X)$ and $M_{ij(X)}$-$\theta O(X)$, respectively.*

Lemma 4 ([13]). *Let $(X, M_{1(X)}, M_{2(X)})$ be a biminimal space. If $S \in M_{j(X)}$ then $M_{ij(X)}$-$Cl_\theta(S) = M_{i(X)}$-$Cl(S)$.*

Lemma 5 ([13]). *Let $(X, M_{1(X)}, M_{2(X)})$ be a biminimal space and $S \subset X$. Then*

(a) $X \setminus M_{ij(X)}$-$Cl_\theta(S) = M_{ij(X)}$-$Int_\theta(X \setminus S)$;

(b) $X \setminus M_{ij(X)}$-$Int_\theta(S) = M_{ij(X)}$-$Cl_\theta(X \setminus S)$.

2 Weakly M_{ij}-preopen functions

Definition 7. *A function $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is said to be weakly M_{ij}-preopen if $g(S) \subseteq M_{ij(Y)}\text{-}Int_p(g(M_{j(X)} - Cl(S)))$ for every $S \in M_{i(X)}$.*

Theorem 1. *Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be a function. Then the results stated below are equivalent:*

(a) g is weakly M_{ij}-preopen;

(b) $g(M_{ij(X)}\text{-}Int_\theta(S)) \subset M_{ij(Y)}\text{-}Int_p(g(S))$ for every $S \subset X$;

(c) $M_{ij(X)}\text{-}Int_\theta(g^{-1}(T)) \subset g^{-1}(M_{ij(Y)}\text{-}Int_p(T))$ for every $T \subset Y$;

(d) $g^{-1}(M_{ij(Y)}\text{-}Cl_p(T)) \subset M_{ij(X)}\text{-}Cl_\theta(g^{-1}(T))$ for every $T \subset Y$;

(e) for every $a \in X$ and for every $F \in M_{i(X)}$ containing a there exists $G \in M_{ij(Y)}\text{-}PO(Y)$ containing $g(a)$ such that $G \subset g(M_{j(X)}\text{-}Cl(F))$.

Proof. (a) $\Rightarrow$ (b). Let $S \subset X$ and $a \in M_{ij(X)}\text{-}Int_\theta(S)$. So there exists $F \in M_{i(X)}$ such that $a \in F \subset M_{j(X)}\text{-}Cl(F) \subset S$. Thus $g(a) \in g(F) \subset g(M_{j(X)}\text{-}Cl(F)) \subset g(S)$. By *(a)*, we have $g(F) \subset M_{ij(Y)}\text{-}Int_p(g(M_{j(X)}\text{-}Cl(F))) \subset M_{ij(Y)}\text{-}Int_p(g(S))$. So, $g(a) \in M_{ij(Y)}\text{-}Int_p(g(S))$. This implies that $a \in g^{-1}(M_{ij(Y)}\text{-}Int_p(g(S)))$. Thus $M_{ij(X)}\text{-}Int_\theta(S) \subset g^{-1}(M_{ij(Y)}\text{-}Int_p(g(S)))$. Hence, $g(M_{ij(X)}\text{-}Int_\theta(S)) \subset M_{ij(Y)}\text{-}Int_p(g(S))$.

(b) $\Rightarrow$ (c). Let $T \subset Y$. Then $g^{-1}(T) \subset X$. By *(b)*, we have $g(M_{ij(X)}\text{-}Int_\theta(g^{-1}(T))) \subset M_{ij(Y)}\text{-}Int_p(g(g^{-1}(T))) \subset M_{ij(Y)}\text{-}Int_p(T)$. Hence, $M_{ij(X)}\text{-}Int_\theta(g^{-1}(T)) \subset g^{-1}(M_{ij(Y)}\text{-}Int_p(T))$.

(c) $\Rightarrow$ (d). Let $T \subset Y$ and $a \notin M_{ij(X)}\text{-}Cl_\theta(g^{-1}(T))$. So,

$$\begin{aligned} a \in X \setminus M_{ij(X)}\text{-}Cl_\theta(g^{-1}(T)) &= M_{ij(X)}\text{-}Int_\theta(X \setminus g^{-1}(T)) \\ &= M_{ij(X)}\text{-}Int_\theta(g^{-1}(Y \setminus T)) \subset g^{-1}(M_{ij(Y)}\text{-}Int_p(Y \setminus T)) \\ &= g^{-1}(Y \setminus M_{ij(Y)}\text{-}Cl_p(T)) = X \setminus g^{-1}(M_{ij(Y)}\text{-}Cl_p(T)). \end{aligned}$$

This implies $a \notin g^{-1}(M_{ij(Y)}\text{-}Cl_p(T))$. Hence, $g^{-1}(M_{ij(Y)}\text{-}Cl_p(T)) \subset M_{ij(X)}\text{-}Cl_\theta(g^{-1}(T))$.

(d) $\Rightarrow$ (e). Let $a \in X$ and $F \in M_{i(X)}$ containing a. Suppose that, $T = Y \setminus g(M_{j(X)}\text{-}Cl(F))$. Then by *(d)* and Lemma 4, we have

$$\begin{aligned} & g^{-1}\Big(M_{ij(Y)}\text{-}Cl_p(Y \setminus g(M_{j(X)}\text{-}Cl(F)))\Big) \subset M_{ij(X)}\text{-}Cl_\theta\Big(g^{-1}(Y \setminus g(M_{j(X)}\text{-}Cl(F)))\Big) \\ &= M_{ij(X)}\text{-}Cl_\theta\Big(X \setminus g^{-1}(g(M_{j(X)}\text{-}Cl(F)))\Big) \subset M_{ij(X)}\text{-}Cl_\theta(X \setminus M_{j(X)}\text{-}Cl(F)) \\ &= M_{i(X)}\text{-}Cl(X \setminus M_{j(X)}\text{-}Cl(F)) = X \setminus M_{i(X)}\text{-}Int(M_{j(X)}\text{-}Cl(F)) \subset X \setminus M_{i(X)}\text{-}Int(F) = X \setminus F. \end{aligned}$$

This implies that,

$$\begin{aligned} & g^{-1}\Big(M_{ij(Y)}\text{-}Cl_p(Y \setminus g(M_{j(X)}\text{-}Cl(F)))\Big) \subset X \setminus F \quad \Rightarrow \\ & g^{-1}\Big(Y \setminus M_{ij(Y)}\text{-}Int_p(g(M_{j(X)}\text{-}Cl(F)))\Big) \subset X \setminus F \quad \Rightarrow \\ & X \setminus g^{-1}\Big(M_{ij(Y)}\text{-}Int_p(g(M_{j(X)}\text{-}Cl(F)))\Big) \subset X \setminus F. \end{aligned}$$

Thus, $F \subset g^{-1}\big(M_{ij(Y)}\text{-}Int_p\big(g(M_{j(X)}\text{-}Cl(F))\big)\big)$ and so

$$g(a) \in g(F) \subset M_{ij(Y)}\text{-}Int_p\big(g\big(M_{j(X)}\text{-}Cl(F)\big)\big) \subset g\big(M_{j(X)}\text{-}Cl(F)\big).$$

Let $G = M_{ij(Y)}\text{-}Int_p\big(g(M_{j(X)}\text{-}Cl(F))\big)$. Then $G \in M_{ij(Y)}\text{-}PO(Y)$ and $g(a) \in G \subset g\big(M_{j(X)}\text{-}Cl(F)\big)$.

(e) ⇒ (a). Let $F \in M_{i(X)}$ containing a. By *(e)*, there exists $G \in M_{ij(Y)}\text{-}PO(Y)$ containing $g(a)$ such that $G \subset g\big(M_{j(X)}\text{-}Cl(F)\big)$. So,

$$g(a) \in G = M_{ij(Y)}\text{-}Int_p(G) \subset M_{ij(Y)}\text{-}Int_p\big(g\big(M_{j(X)}\text{-}Cl(F)\big)\big).$$

Consequently, $g(F) \subset M_{ij(Y)}\text{-}Int_p\big(g(M_{j(X)} - Cl(F))\big)$ and hence g is weakly M_{ij}-preopen function. □

Theorem 2. *Let $g : \big(X, M_{1(X)}, M_{2(X)}\big) \to \big(Y, M_{1(Y)}, M_{2(Y)}\big)$ be a function. Then the results stated below are equivalent:*

(a) g is weakly M_{ij}-preopen;

(b) $g\big(M_{i(X)}\text{-}Int(S)\big) \subset M_{ij(Y)}\text{-}Int_p\big(g(S)\big)$ for every $M_{j(X)}$-closed set S of X;

(c) $g(T) \subset M_{ij(Y)}\text{-}Int_p\big(g(M_{j(X)}\text{-}Cl(T))\big)$ for every $T \in M_{ij(X)}\text{-}PO(X)$;

(d) $g(T) \subset M_{ij(Y)}\text{-}Int_p\big(g(M_{j(X)}\text{-}Cl(T))\big)$ for every $T \in M_{ij(X)}\text{-}\alpha O(X)$.

Proof. *(a) ⇒ (b).* Let S be $M_{j(X)}$-closed set in X. So, $S = M_{j(X)}\text{-}Cl(S)$. Since $M_{i(X)}\text{-}Int(S) \in M_{i(X)}$, so by *(a)* we have

$$\begin{aligned} g\big(M_{i(X)}\text{-}Int(S)\big) &\subset M_{ij(Y)}\text{-}Int_p\Big(g\big(M_{j(X)}\text{-}Cl(M_{i(X)}\text{-}Int(S))\big)\Big) \\ &\subset M_{ij(Y)}\text{-}Int_p\big(g(M_{j(X)}\text{-}Cl(S))\big) = M_{ij(Y)}\text{-}Int_p\big(g(S)\big). \end{aligned}$$

Hence, $g\big(M_{i(X)}\text{-}Int(S)\big) \subset M_{ij(Y)}\text{-}Int_p\big(g(S)\big)$.

(b) ⇒ (c). Let $T \in M_{ij(X)}\text{-}PO(X)$. Then $T \subset M_{i(X)}\text{-}Int\big(M_{j(X)}\text{-}Cl(T)\big)$. This implies $g(T) \subset g\big(M_{i(X)}\text{-}Int\big(M_{j(X)}\text{-}Cl(T)\big)\big)$. Since $M_{j(X)}\text{-}Cl(T)$ is $M_{j(X)}$-closed, so by *(b)* we have

$$g(T) \subset g\big(M_{i(X)}\text{-}Int\big(M_{j(X)}\text{-}Cl(T)\big)\big) \subset M_{ij(Y)}\text{-}Int_p\big(g\big(M_{j(X)}\text{-}Cl(T)\big)\big).$$

Hence, $g(T) \subset M_{ij(Y)}\text{-}Int_p\big(g\big(M_{j(X)}\text{-}Cl(T)\big)\big)$.

(c) ⇒ (d). Let $T \in M_{ij(X)}\text{-}\alpha O(X)$. Then

$$T \subset M_{i(X)}\text{-}Int\big(M_{j(X)}\text{-}Cl\big(M_{i(X)}\text{-}Int(T)\big)\big) \subset M_{i(X)}\text{-}Int\big(M_{j(X)}\text{-}Cl(T)\big).$$

Thus $T \in M_{ij(X)}\text{-}PO(X)$. Hence by *(c)*, the result follows.

(d) ⇒ (a). Let $T \in M_{i(X)}$. Then

$$T = M_{i(X)}\text{-}Int(T) \subset M_{i(X)}\text{-}Int\big(M_{j(X)}\text{-}Cl(T)\big) = M_{i(X)}\text{-}Int\big(M_{j(X)}\text{-}Cl\big(M_{i(X)}\text{-}Int(T)\big)\big).$$

That is, $T \subset M_{i(X)}\text{-}Int\big(M_{j(X)}\text{-}Cl\big(M_{i(X)}\text{-}Int(T)\big)\big)$. This implies $T \in M_{ij(X)}\text{-}\alpha O(X)$. Now by *(d)*, we have $g(T) \subset M_{ij(Y)}\text{-}Int_p\big(g\big(M_{j(X)}\text{-}Cl(T)\big)\big)$. Hence, g is weakly M_{ij}-preopen function. □

Theorem 3. *Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be a function. Then the results stated below are equivalent:*

(a) g is weakly M_{ij}-preopen;

(b) $M_{ij(Y)}\text{-}Cl_p\Big(g\big(M_{j(X)}\text{-}Int\big(M_{i(X)}\text{-}Cl(S)\big)\big)\Big) \subset g\big(M_{i(X)}\text{-}Cl(S)\big)$ for every $S \subset X$;

(c) $M_{ij(Y)}\text{-}Cl_p\big(g\big(M_{j(X)}\text{-}Int(T)\big)\big) \subset g(T)$ for every $T \in M_{ij(X)}\text{-}RC(X)$;

(d) $M_{ij(Y)}\text{-}Cl_p\big(g(S)\big) \subset g\big(M_{i(X)}\text{-}Cl(S)\big)$ for every $S \in M_{j(X)}$.

Proof. (a) $\Rightarrow$ (b). Let $a \in X$ and $S \subset X$. Also, let $g(a) \in Y \setminus g\big(M_{i(X)}\text{-}Cl(S)\big)$. This implies $a \in X \setminus M_{i(X)}\text{-}Cl(S)$ and so there exists $T \in M_{i(X)}$ containing a such that $T \cap S = \varnothing$. Thus, $M_{j(X)}\text{-}Cl(T) \cap M_{j(X)}\text{-}Int\big(M_{i(X)}\text{-}Cl(S)\big) = \varnothing$. Since g is weakly M_{ij}-preopen, so by Theorem 1, there exists $G \in M_{ij(Y)}\text{-}PO(Y)$ containing $g(a)$ such that $G \subset g\big(M_{j(X)}\text{-}Cl(T)\big)$.

So, $G \cap g\big(M_{j(X)}\text{-}Int\big(M_{i(X)}\text{-}Cl(S)\big)\big) = \varnothing$. Then

$$g(a) \in Y \setminus M_{ij(Y)}\text{-}Cl_p\Big(g\big(M_{j(X)}\text{-}Int\big(M_{i(X)}\text{-}Cl(S)\big)\big)\Big).$$

Hence, $M_{ij(Y)}\text{-}Cl_p\Big(g\big(M_{j(X)}\text{-}Int\big(M_{i(X)}\text{-}Cl(S)\big)\big)\Big) \subset g\big(M_{i(X)}\text{-}Cl(S)\big)$.

(b) $\Rightarrow$ (c). Let $T \in M_{ij(X)}\text{-}RC(X)$. So, $T = M_{i(X)}\text{-}Cl\big(M_{j(X)}\text{-}Int(T)\big)$. Now we see that

$$\begin{aligned} M_{ij(Y)}\text{-}Cl_p\Big(g\big(M_{j(X)}\text{-}Int(T)\big)\Big) &= M_{ij(Y)}\text{-}Cl_p\Big(g\big(M_{j(X)}\text{-}Int\big(M_{i(X)}\text{-}Cl\big(M_{j(X)}\text{-}Int(T)\big)\big)\big)\Big) \\ &\subset g\big(M_{i(X)}\text{-}Cl\big(M_{j(X)}\text{-}Int(T)\big)\big) = g(T). \end{aligned}$$

Consequently, $M_{ij(Y)}\text{-}Cl_p\big(g\big(M_{j(X)}\text{-}Int(T)\big)\big) \subset g(T)$.

(c) $\Rightarrow$ (d). Let $S \in M_{j(X)}$. Then $S = M_{j(X)}\text{-}Int(S)$. Now,

$$M_{i(X)}\text{-}Cl(S) = M_{i(X)}\text{-}Cl(M_{j(X)}\text{-}Int(S)) \subset M_{i(X)}\text{-}Cl\big(M_{j(X)}\text{-}Int\big(M_{i(X)}\text{-}Cl(S)\big)\big).$$

Also, $M_{j(X)}\text{-}Int\big(M_{i(X)}\text{-}Cl(S)\big) \subset M_{i(X)}\text{-}Cl(S)$ implies

$$M_{i(X)}\text{-}Cl\big(M_{j(X)}\text{-}Int\big(M_{i(X)}\text{-}Cl(S)\big)\big) \subset M_{i(X)}\text{-}Cl\big(M_{i(X)}\text{-}Cl(S)\big) = M_{i(X)}\text{-}Cl(S).$$

Thus, $M_{i(X)}\text{-}Cl(S) = M_{i(X)}\text{-}Cl\big(M_{j(X)}\text{-}Int\big(M_{i(X)}\text{-}Cl(S)\big)\big)$ and so $M_{i(X)}\text{-}Cl(S) \in M_{ij(X)}\text{-}RC(X)$. Now, by *(c)* we have

$$\begin{aligned} M_{ij(Y)}\text{-}Cl_p\big(g(S)\big) &= M_{ij(Y)}\text{-}Cl_p\big(g\big(M_{j(X)}\text{-}Int(S)\big)\big) \\ &\subset M_{ij(Y)}\text{-}Cl_p\big(g\big(M_{j(X)}\text{-}Int\big(M_{i(X)}\text{-}Cl(S)\big)\big)\big) \subset g\big(M_{i(X)}\text{-}Cl(S)\big). \end{aligned}$$

Hence, $M_{ij(Y)}\text{-}Cl_p\big(g(S)\big) \subset g\big(M_{i(X)}\text{-}Cl(S)\big)$.

(d) $\Rightarrow$ (a). Let $S \in M_{i(X)}$. Then $M_{j(X)}\text{-}Cl(S)$ is $M_{j(X)}$-closed and $X \setminus M_{j(X)}\text{-}Cl(S) \in M_{j(X)}$. Now, by *(d)* we have

$$\begin{aligned} Y \setminus M_{ij(Y)}\text{-}Int_p\big(g\big(M_{j(X)}\text{-}Cl(S)\big)\big) &= M_{ij(Y)}\text{-}Cl_p\big(Y \setminus g\big(M_{j(X)}\text{-}Cl(S)\big)\big) \\ &= M_{ij(Y)}\text{-}Cl_p\big(g\big(X \setminus M_{j(X)}\text{-}Cl(S)\big)\big) \\ &\subset g\big(M_{i(X)}\text{-}Cl\big(X \setminus M_{j(X)}\text{-}Cl(S)\big)\big) \\ &= g\big(X \setminus M_{i(X)}\text{-}Int\big(M_{j(X)}\text{-}Cl(S)\big)\big) \\ &\subset g\big(X \setminus M_{i(X)}\text{-}Int(S)\big) \\ &= g(X \setminus S) = Y \setminus g(S). \end{aligned}$$

Hence, $g(S) \subset M_{ij(Y)}\text{-}Int_p\big(g\big(M_{j(X)}\text{-}Cl(S)\big)\big)$ and so g is weakly M_{ij}-preopen function. □

Theorem 4. *Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be a function. Then the results stated below are equivalent:*

(a) g is weakly M_{ij}-preopen;

(b) $g^{-1}(M_{ij(Y)}\text{-}Cl_p(T)) \subset M_{ij(X)}\text{-}Cl_\theta(g^{-1}(T))$ for every $T \subset Y$;

(c) $M_{ij(Y)}\text{-}Cl_p(g(S)) \subset g(M_{ij(X)}\text{-}Cl_\theta(S))$ for every $S \subset X$;

(d) $M_{ij(Y)}\text{-}Cl_p(g(M_{j(X)}\text{-}Int(M_{ij(X)}\text{-}Cl_\theta(S)))) \subset g(M_{ij(X)}\text{-}Cl_\theta(S))$ for every $S \subset X$.

Proof. *(a) ⇒ (b).* Let $T \subset Y$ and $a \in g^{-1}(M_{ij(Y)}\text{-}Cl_p(T))$. This implies $g(a) \in M_{ij(Y)}\text{-}Cl_p(T)$. Also let $F \in M_{i(X)}$ such that $a \in F$. Since g is weakly M_{ij}-preopen, so by Theorem 1, there exists $G \in M_{ij(Y)}\text{-}PO(X)$ containing $g(a)$ such that $G \subset g(M_{j(X)}\text{-}Cl(F))$. Since $g(a) \in M_{ij(Y)}\text{-}Cl_p(T)$, so $G \cap T \neq \varnothing$ and hence $\varnothing \neq g^{-1}(G) \cap g^{-1}(T) \subset M_{j(X)}\text{-}Cl(F) \cap g^{-1}(T)$.

This implies $M_{j(X)}\text{-}Cl(F) \cap g^{-1}(T) \neq \varnothing$ and so $a \in M_{ij(X)}\text{-}Cl_\theta(g^{-1}(T))$. Hence, *(b)* holds.

(b) ⇒ (c). Let $S \subset X$. Then $g(S) \subset Y$. By *(b)*, we have

$$g^{-1}(M_{ij(Y)}\text{-}Cl_p(g(S))) \subset M_{ij(X)}\text{-}Cl_\theta(g^{-1}(g(S))) \subset M_{ij(X)}\text{-}Cl_\theta(S).$$

Hence, $M_{ij(Y)}\text{-}Cl_p(g(S)) \subset g(M_{ij(X)}\text{-}Cl_\theta(S))$.

(c) ⇒ (d). Let $S \subset X$. Since $M_{ij(X)}\text{-}Cl_\theta(S)$ is $M_{i(X)}$-closed, so $M_{i(X)}\text{-}Cl(M_{ij(X)}\text{-}Cl_\theta(S)) = M_{ij(X)}\text{-}Cl_\theta(S)$. Also, $M_{j(X)}\text{-}Int(M_{ij(X)}\text{-}Cl_\theta(S)) \in M_{j(X)}$. By *(c)* and Lemma 4, we have

$$\begin{aligned} M_{ij(Y)}\text{-}Cl_p\Big(g(M_{j(X)}\text{-}Int(M_{ij(X)}\text{-}Cl_\theta(S)))\Big) &\subset g\Big(M_{ij(X)}\text{-}Cl_\theta(M_{j(X)}\text{-}Int(M_{ij(X)}\text{-}Cl_\theta(S)))\Big) \\ &= g\Big(M_{i(X)}\text{-}Cl(M_{j(X)}\text{-}Int(M_{ij(X)}\text{-}Cl_\theta(S)))\Big) \\ &\subset g(M_{i(X)}\text{-}Cl(M_{ij(X)}\text{-}Cl_\theta(S))) \\ &= g(M_{ij(X)}\text{-}Cl_\theta(S)). \end{aligned}$$

Hence, $M_{ij(Y)}\text{-}Cl_p(g(M_{j(X)}\text{-}Int(M_{ij(X)}\text{-}Cl_\theta(S)))) \subset g(M_{ij(X)}\text{-}Cl_\theta(S))$.

(d) ⇒ (a). Let $G \in M_{j(X)}$. Then by Lemma 4 we have

$$G = M_{j(X)}\text{-}Int(G) \subset M_{j(X)}\text{-}Int(M_{i(X)}\text{-}Cl(G)) = M_{j(X)}\text{-}Int(M_{ij(X)}\text{-}Cl_\theta(G)).$$

Using *(d)* and Lemma 4, from the above one can obtain

$$\begin{aligned} M_{ij(Y)}\text{-}Cl_p(g(G)) &\subset M_{ij(Y)}\text{-}Cl_p\Big(g(M_{j(X)}\text{-}Int(M_{ij(X)}\text{-}Cl_\theta(G)))\Big) \\ &\subset g(M_{ij(X)} - Cl_\theta(S)) \\ &= g(M_{i(X)} - Cl(S)). \end{aligned}$$

Thus, $M_{ij(Y)}\text{-}Cl_p(g(G)) \subset g(M_{i(X)}\text{-}Cl(S))$. Hence by Theorem 3, g is weakly M_{ij}-preopen function. □

Theorem 5. *A function $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is weakly M_{ij}-preopen if $g(M_{ij(X)}\text{-}Cl_\theta(S))$ is $M_{ij(Y)}$-preclosed in Y for every subset S of X.*

Proof. Let $S \subset X$ and $g(M_{ij(X)}\text{-}Cl_\theta(S))$ be $M_{ij(Y)}$-preclosed in Y. Then

$$M_{ij(Y)}\text{-}Cl_p(g(S)) \subset M_{ij(Y)}\text{-}Cl_p(g(M_{ij(X)}\text{-}Cl_\theta(S))) = g(M_{ij(X)}\text{-}Cl_\theta(S)).$$

Thus $M_{ij(Y)}\text{-}Cl_p(g(S)) \subset g(M_{ij(X)}\text{-}Cl_\theta(S))$. Hence, by Theorem 4, we have g is weakly M_{ij}-preopen function. □

Theorem 6. *If $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is weakly M_{ij}-preopen function and $g(M_{j(X)}\text{-}Cl(S)) \subset g(S)$ for every $S \in M_{i(X)}$, then $g(S) \in M_{ij(Y)}\text{-}PO(Y)$.*

Proof. Let $S \in M_{i(X)}$. Since g is weakly M_{ij}-preopen, so

$$g(S) \subset M_{ij(Y)}\text{-}Int_p(g(M_{j(X)}\text{-}Cl(S))) \subset M_{ij(Y)}\text{-}Int_p(g(S)).$$

Also, we have $M_{ij(Y)}\text{-}Int_p(g(S)) \subset g(S)$. Thus $g(S) = M_{ij(Y)}\text{-}Int_p(g(S))$ and so $g(S) \in M_{ij(Y)}\text{-}PO(Y)$. □

Definition 8. *A biminimal space $(X, M_{1(X)}, M_{2(X)})$ is said to be $M_{ij(X)}$-regular if for every $a \in X$ and for every $S \in M_{i(X)}$ containing a, there exists $T \in M_{i(X)}$ such that*

$$a \in T \subset M_{j(X)}\text{-}Cl(T) \subset S.$$

Theorem 7. *Let $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ be a function such that the biminimal space $(X, M_{1(X)}, M_{2(X)})$ is $M_{ij(X)}$-regular. Then the results stated below are equivalent:*

(a) g is weakly M_{ij}-preopen;

(b) $g(S) \in M_{ij(Y)}\text{-}PC(Y)$ for all $S \in M_{ij(X)}\text{-}\theta C(X)$;

(c) $g(T) \in M_{ij(Y)}\text{-}PO(Y)$ for all $T \in M_{ij(X)}\text{-}\theta O(X)$;

(d) for every $S \in M_{ij(X)}\text{-}\theta C(X)$ and for every subset T of Y such that $g^{-1}(T) \subset S$, there exists $Q \in M_{ij(Y)}\text{-}PC(Y)$ containing T such that $g^{-1}(Q) \subset S$.

Proof. (a) ⇒ (b). Let $S \in M_{ij(X)}\text{-}\theta C(X)$. Since g is weakly M_{ij}-preopen, so by Theorem 4 we have $M_{ij(Y)}\text{-}Cl_p(g(S)) \subset g(M_{ij(X)}\text{-}Cl_\theta(S)) = g(S)$. Since $g(S) \subset M_{ij(Y)}\text{-}Cl_p(g(S))$, so $g(S) = M_{ij(Y)}\text{-}Cl_p(g(S))$ and hence $g(S) \in M_{ij(Y)}\text{-}PC(Y)$.

(b) ⇒ (c). Let $T \in M_{ij(X)}\text{-}\theta O(X)$. Then $X \setminus T \in M_{ij(X)}\text{-}\theta C(X)$. By *(b)*, we have $g(X \setminus T) = Y \setminus g(T) \in M_{ij(Y)}\text{-}PC(Y)$. Hence $g(T) \in M_{ij(Y)}\text{-}PO(Y)$.

(c) ⇒ (d). Let $T \subset Y$ and $S \in M_{ij(X)}\text{-}\theta C(X)$ be such that $g^{-1}(T) \subset S$. Since we have $X \setminus S \in M_{ij(X)}\text{-}\theta O(X)$, so by *(c)*, we obtain $g(X \setminus S) \in M_{ij(Y)}\text{-}PO(Y)$. Let $Q = Y \setminus g(X \setminus S)$. Then $Q \in M_{ij(Y)}\text{-}PC(Y)$. Now, $g^{-1}(T) \subset S \Rightarrow X \setminus S \subset X \setminus g^{-1}(T) \Rightarrow g(X \setminus S) \subset Y \setminus T \Rightarrow T \subset Y \setminus g(X \setminus S) = Q$. Also, $g^{-1}(Q) = g^{-1}(Y \setminus g(X \setminus S)) = g^{-1}(g(S)) \subset S$. Thus, there exists $Q \in M_{ij(Y)}\text{-}PC(Y)$ containing T such that $g^{-1}(Q) \subset S$.

(d) ⇒ (a). Let $T \subset Y$ and $S = M_{ij(X)}\text{-}Cl_\theta(g^{-1}(T))$. Since $(X, M_{1(X)}, M_{2(X)})$ is $M_{ij(X)}$-regular, so $S \in M_{ij(X)}\text{-}\theta C(X)$ and $g^{-1}(T) \subset S$. By *(d)*, there exists $Q \in M_{ij(Y)}\text{-}PC(Y)$ containing T such that $g^{-1}(Q) \subset S$. Since $Q \in M_{ij(Y)}\text{-}PC(Y)$, so

$$g^{-1}(M_{ij(Y)}\text{-}Cl_p(T)) \subset g^{-1}(Q) \subset S = M_{ij(X)}\text{-}Cl_\theta(g^{-1}(T)).$$

Hence, by Theorem 4, g is weakly M_{ij}-preopen function. □

Definition 9. *A function $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is said to be contra M_{ij}-preopen (respectively, contra M_{ij}-preclosed) if $g(S) \in M_{ij(Y)}$-$PC(Y)$ (respectively, $g(S) \in M_{ij(Y)}$-$PO(Y)$) for every $S \in M_{j(X)}$ (respectively, for every $X \setminus S \in M_{j(X)}$).*

Theorem 8. *If $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is a contra M_{ij}-preclosed function, then g is weakly M_{ij}-preopen.*

Proof. Let $S \in M_{i(X)}$. Then $M_{j(X)}$-$Cl(S)$ is $M_{j(X)}$-closed in X. Since g is contra M_{ij}-preclosed and $M_{j(X)}$-$Cl(S)$ is $M_{j(X)}$-closed in X, so $g(M_{j(X)}\text{-}Cl(S)) \in M_{ij(Y)}$-$PO(Y)$. Therefore, $g(S) \subset g(M_{j(X)}\text{-}Cl(S)) = M_{ij(Y)}\text{-}Int_p(g(M_{j(X)}\text{-}Cl(S)))$. Hence, g is weakly M_{ij}-preopen function. □

Theorem 9. *If $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is a contra M_{ij}-preopen function, then g is weakly M_{ij}-preopen.*

Proof. Let $S \in M_{j(X)}$. Then $M_{i(X)}$-$Cl(S)$ is $M_{i(X)}$-closed in X. Since g is contra M_{ij}-preopen, so $g(S) \in M_{ij(Y)}$-$PC(Y)$. Therefore, $M_{ij(Y)}\text{-}Cl_p(g(S)) = g(S) \subset g(M_{i(X)}\text{-}Cl(S))$. So, by Theorem 3, g is weakly M_{ij}-preopen function. □

Theorem 10. *Let $(X, M_{1(X)}, M_{2(X)})$ and $(Y, M_{1(Y)}, M_{2(Y)})$ be two biminimal spaces such that $M_{j(X)}\text{-}Cl(S) = X$ for every $S \in M_{i(X)}$. Then a function $g : (X, M_{1(X)}, M_{2(X)}) \to (Y, M_{1(Y)}, M_{2(Y)})$ is weakly M_{ij}-preopen if and only if $g(X) \in M_{ij(Y)}$-$PO(Y)$.*

Proof. Let g be a weakly M_{ij}-preopen function. Since $X \in M_{i(X)}$, so

$$g(X) \subset M_{ij(Y)}\text{-}Int_p(g(M_{j(X)}\text{-}Cl(X))) = M_{ij(Y)}\text{-}Int_p(g(X)).$$

Therefore $g(X) \in M_{ij(Y)}$-$PO(Y)$.

Conversely, let $g(X) \in M_{ij(Y)}$-$PO(Y)$. Also let $S \in M_{i(X)}$. Then

$$g(S) \subset g(X) = M_{ij(Y)}\text{-}Int_p(g(X)) = M_{ij(Y)}\text{-}Int_p(g(M_{j(X)}\text{-}Cl(S))).$$

Thus, g is weakly M_{ij}-preopen function. □

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Метою цієї статті є визначення поняття слабко M-передвідкритої функції в бімінімальних структурних просторах. Було встановлено кілька властивостей такої функції та досліджено її зв'язок з деякими іншими поняттями, пов'язаними з M-передвідкритими множинами в бімінімальних просторах.

Ключові слова і фрази: бімінімальний простір, $M_{ij(X)}$-передвідкрита множина, $M_{ij(X)}$-передзамкнута множина, $M_{ij(X)}$-θ-відкрита множина, $M_{ij(X)}$-θ-замкнута множина, M_{ij}-слабко передвідкрита функція.