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On the Hypersonic Absorption in Straight-line Graphene Ribbons

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We have investigated the intraband and interband electron mechanisms of hypersonic absorption in straight-line graphene ribbons. The absorption coefficients of confined hypersonic phonons, taking into account the screening of deformative electron-phonon interaction with charge carriers and the dispersion laws of Dirac electron states, have been determined.

Keywords: straight-line graphene wire, confined acoustic phonons, intraband and interband electron transitions, hypersonic absorption.

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Introduction

One of outstanding experimental achievements of the recent past was obtaining graphene [1,2] and discovering its physical properties, so strange and interesting for modern nanoelectronics. Graphene is a one-atom thick plane of graphite. A gap between the valence band and the conduction band of graphene is equal to zero, as well as effective mass of electrons and holes. Therefore graphene electrons and holes are described by Dirac's equation, but their mass is equal to zero (like that one of neutrinos). Most of research works are dedicated to electrical and optical properties of infinite graphene. The electronic states in confined graphene ribbons are researched in the works [3-5]. The work [5] studies various models of graphene wire.

Whereas acoustic methods are widely used in solid state physics, the role of quantum acoustics [6-8], associated with high-frequency hypersonic waves spread in mesoscopic and nanostructured materials [8], is still growing. The work [9] provides a comparative study of dispersion of graphite and infinite graphene phonons. The work [10] describes acoustic oscillations in the two-dimensional model of straight-line graphene ribbons with calculated dispersion curves of confined acoustic phonons of hybrid modes for different widths of graphene quantum wire. This research work aims at calculating the coefficients of electron absorption of the lowest mode of hypersound during intraband and interband transitions in straight-line graphene wire.

I. Confined acoustic phonons and electrons in straight-line graphene ribbon

We consider the model of flat graphene wire, infinitely long in the direction of X-axis and 2d wide in the direction of Y-axis $(-L/2 \le x \le L/2, -d \le y \le d, L$? 2d). The lowest mode of confined acoustic phonons with frequency $\omega_{\gamma} = c_{\gamma}\gamma$ and wave vector γ in the direction of X-axis of waveguide is defined from this dispersion equation [10]:

$$\frac{\mathrm{tg}(\pi\sqrt{X-\Psi})}{\mathrm{tg}(\pi\sqrt{\delta X-\Psi})} = -\frac{\Psi\sqrt{X-\Psi}\sqrt{\delta X-\Psi}}{\left(0.5X-\Psi\right)^2},\qquad(1)$$

where

$$\omega_{\gamma} = \frac{\pi c_{s}}{d} \sqrt{X} , \ \gamma = \frac{\pi}{d} \sqrt{\Psi} , \qquad (2)$$

$$\delta = (c_s / c_d)^2 = (1 - 2\sigma) / 2(1 - \sigma), \qquad (3)$$

 c_d and c_s – velocity of longitudinal and transverse sound in the infinite graphene. The dispersion curves of the lowest acoustic mode are calculated for various widths of wire 2d = 20Å, 40Å, 80Å, 100Å (shown in Figure 1), using the data of [9] for infinite graphene: $c_s = 1.28 \cdot 10^6$ cm/s, $c_d = 2.12 \cdot 10^6$ cm/s, $\delta = 0.36$, $\sigma \approx 0.22$. The quantum-size limits, defined for acoustic phonons in straight-line graphene ribbon, are related to longitudinal and transverse atomic displacements and, at



Fig. 1. Dispersion curves of the lowest mode of acoustic phonons of straight-line graphene wire for width: $1 - 20\text{\AA}$, $2 - 40\text{\AA}$, $3 - 80\text{\AA}$, $4 - 100\text{\AA}$.

the same time, to significant dependence of phonon spectrum on ribbon width, if γ is rather high.

The work [5] shows bound states, existing due to one-dimensional symmetric potential well, on basis of Dirac's equation with zero effective mass of an electron: $w(y) = -w_0$ if $|y| \le d$ and w(y) = 0 if y < -d, y > d ($w_0 > 0$). Any bound state is defined by a two-component spinor in the direction of Y-axis; as for the direction of X-axis, translational motion of an electron with wave vector k is typical for it. In [5] analytical expressions for the energy spectrum of Dirac bound electronic states in straight-line graphene ribbon are obtained for three significant limiting cases. The first

case corresponds to the linear dispersion law $E_1(k) = \mathbf{h} |k| u |\cos(2q_0 d)|$ if $|k| = q_0$, $\cos(2q_0 d) \neq 0$, where $q_0 = w_0 / \mathbf{h} u$, $u \approx 10^8 \text{ cm/s}$ is graphene-specific velocity. Provided that $|k| = q_0$, $\cos(2q_0 d) = 0$, we have the quadratic dispersion law $E_2(k) = \mathbf{h}^2 k^2 u^2 / w_0$. Under the condition of small depth of well and width of ribbon $w_0 \rightarrow 0$ and $d \rightarrow 0$, the dispersion law becomes linear again: $E_3(k) = \mathbf{h} |k| u [1 - 2(q_0 d)^2]$. The normalized wave function of a conduction electron with energy E_k is as follows:

$$\Psi_{k,s}(\mathbf{x},\mathbf{y}) = \frac{1}{\sqrt{L}} e^{ikx} \begin{pmatrix} s\Psi_{1s}(\mathbf{y}) \\ \Psi_{2s}(\mathbf{y}) \end{pmatrix}, \tag{4}$$

where s = +1 corresponds to the conduction band, and s = -1 corresponds to the valence band; $\Psi_{1s}(y) = A_{1s}e^{\chi_{s}y}$ if y < -d, $\Psi_{1s}(y) = A_{3}e^{-\chi_{s}y}$ if y > d, $\Psi_{1s} = X_{1s}(y)$ if $|y| \le d$, $\chi_{s} = \sqrt{k^{2} - (E_{k,s} / hu)^{2}}$, $\Psi_{1s}(y) = -\frac{hu}{2} \left(\frac{\partial \Psi_{1s}}{\partial x} + k\Psi_{1s} \right)$ (5)

$$\Psi_{2s}(\mathbf{y}) = \frac{\mathbf{hu}}{\mathbf{E}_{k,s} + \mathbf{w}_0} \left(\frac{\partial \mathbf{I}_{1s}}{\partial \mathbf{y}} + \mathbf{k} \Psi_{1s} \right), \tag{5}$$

$$X_{1s}(y) = B_{1s}\sin(q_s y) + B_{2s}\cos(q_s y), \qquad (6)$$

$$\mathbf{X}_{2s}(\mathbf{y}) = \frac{\mathbf{hu}}{\mathbf{E}_{k,s} + \mathbf{w}_0} \left(\frac{\partial \mathbf{X}_{1s}}{\partial \mathbf{y}} + \mathbf{k} \mathbf{X}_{1s} \right), \tag{7}$$

 $q_s = \sqrt{[(E_{k,s} + w_0)/hu]^2 - k^2}$. The limiting conditions, satisfied by function $\Psi_1(y)$, are calculated by these equations:

$$\Psi_{1s}(\pm d + 0) = \Psi_{1s}(\pm d - 0), \qquad (8)$$

$$\left[\ln\Psi_{1s}(\pm d+0)\right]' - \left[\ln\Psi_{1s}(\pm d-0)\right]' \pm \frac{W_0}{E_{k,s} + W_0} \left[\ln\Psi_{1s}(\pm d-0)\right]' \pm \frac{W_0 k}{E_{k,s} + W_0} = 0.$$
(9)

In the model of finite wire width, the Hamiltonian of interaction between an electron and the lowest mode of hypersound with frequency ω_{γ} and wave vector γ ,

that is different from zero if $|y| \le d$, can be expressed as:

$$\hat{H} = E_{a} \operatorname{div} \hat{u}(x, y, t) = E_{a} A_{\gamma} \frac{\omega_{\gamma}^{2}}{c_{d}^{2} k_{1}} \cos(k_{1} y) \left[\hat{b}_{\gamma} e^{i\gamma(x - c_{\gamma} t)} + h.c. \right],$$
(10)

where E_a is the deformation potential constant, \hat{u} is quantum displacement vector [10], \hat{b}_{γ} and \hat{b}_{γ}^+ are destruction and creation operators of phonons; $k_1 = \gamma \sqrt{(c_{\gamma} / c_d)^2 - 1}$, A_{γ} is an amplitude of corresponding mode, that is expressed by formula (12) in [10]. Screening of electron-phonon interaction with free charge carriers can be taken into consideration in the usual way [11], including modifications, required

for the flat case of straight-line graphene ribbon.

II. Hypersonic absorption with electrons during intraband transitions in straight-line graphene ribbons

Absorption of high-frequency sound may be consi-

dered as the process of absorbing confined acoustic phonons, spreading along X-axis of wire with wave vector γ , and corresponding to the case $\gamma l ? l$, where **l** is average length of free path of an electron in the direction of the wire axis. Applying Fermi's golden rule

to intraband electronic transitions in the vicinity of the K-point of the Brillouin zone, the Hamiltonian of electron-phonon interaction (10) and spinor components (6), (7), we have the following general expression for the coefficient of hypersonic absorption:

coefficient of electron absorption of hypersonic in

straight-line graphene ribbon, taking into account

$$\boldsymbol{\alpha} = \frac{2\pi}{\mathbf{h}c_{\gamma}} \mathbf{g}_{s} \mathbf{g}_{v} \sum_{\mathbf{k},\mathbf{k}'} \left| \left\langle \mathbf{k}' \right| \hat{\mathbf{H}} \right| \mathbf{k} \right\rangle^{2} \left(\mathbf{f}_{\mathbf{k}} - \mathbf{f}_{\mathbf{k}'} \right) \cdot \delta(\mathbf{E}_{\mathbf{k}'} - \mathbf{E}_{\mathbf{k}} - \mathbf{h}\omega_{\gamma}) , \qquad (11)$$

where g_s and g_v are spin and valley degeneracy according to $(g_s = g_v = 2)$, f_k is the Fermi-Dirac distribution function:

$$|\mathbf{k}\rangle = \frac{1}{\sqrt{L}} e^{i\mathbf{k}\mathbf{x}} \begin{pmatrix} \mathbf{X}_1(\mathbf{y}) \\ \mathbf{X}_2(\mathbf{y}) \end{pmatrix}.$$
(12)

Below we summarize the final results for the

1. Linear dispersion law:

screening of deformative electron-phonon interaction for three important limiting cases, in which analytical expressions [5] are obtained for energy spectrum of Dirac bound electronic states.

 $E_{1}(k) = \mathbf{h} |k| u |\cos(2q_{0}d)|, |k| = q_{0}, \cos(2q_{0}d) \neq 0, c_{\gamma} < \mathbf{W}_{F} = u |\cos(2q_{0}d)|, \gamma d = 1.$

$$\alpha_{1} = \frac{2E_{a}^{2}G_{\gamma 1}(\xi)(\gamma d)^{2}\omega_{\gamma}^{3}}{\mathbf{h}c_{\gamma}u\rho daB_{\gamma}(c_{d}^{2}k_{1})^{2}} \left(1 - \frac{c_{\gamma}^{2}}{\vartheta_{F}^{2}}\right) F(\xi)[f(\mathbf{h}\vartheta_{F} \mid \gamma^{(-)} \mid) - f(\mathbf{h}\vartheta_{F} \mid \gamma^{(+)} \mid)] \cdot 8.686 \text{ dB/cm},$$
(13)

where

$$q^{(-)} = -\frac{\gamma}{2} \left(1 - \frac{c_{\gamma}}{\vartheta_{q_F}} \right), \ \gamma^{(+)} = -\frac{\gamma}{2} \left(1 + \frac{c_{\gamma}}{\vartheta_{q_F}} \right), \ \xi = q_0 d , \tag{14}$$

$$F(\xi) = \frac{(\sin^2 2\xi)(1 - \varphi^{(+)}\varphi^{(-)})^2}{\left|\cos(2\xi)\right|(\sin^2 \xi + \varphi^{(+)^2})(\sin^2 \xi + \varphi^{(-)^2})},$$
(15)

expression B_{γ} in (13) is calculated by formulas (Π 1)-(Π 3) in [12],

$$\varphi^{(\pm)} = \frac{|\cos(2\xi)|\cos\xi + (|\sin(2\xi)| \pm 1)\sin\xi}{|\cos(2\xi)|\cos\xi - (|\sin(2\xi)| \pm 1)\sin\xi},$$
(16)

$$f(E) = \left[exp\left(\frac{E-\mu}{\theta}\right) + 1 \right]^{-1}, \quad \theta = k_B T, \quad G_{\gamma I}(\xi) = \left(\frac{\gamma}{\gamma + \mathcal{R}_{FI}}\right)^2, \quad (17)$$

$$\mathbf{R}_{F_{I}} = \frac{2e^{2} \left(1 - \frac{c_{\gamma}}{\mathbf{R}_{F}}\right)^{2} \gamma^{2} d}{\varepsilon_{L} \mathbf{h}_{F} (\sin^{2} \xi + \varphi^{(-)^{2}})^{2}} \left\{ (1 + \varphi^{(-)^{2}})^{2} \left(\frac{3}{4} + \frac{1}{16} \frac{\sin(4\xi)}{\xi}\right) + \frac{1}{2} [\varphi^{(-)^{2}} + \frac{1}{2} (1 + \varphi^{(-)^{2}})^{2}] \left(1 - \frac{\sin(4\xi)}{4\xi}\right) \right\},$$
(18)

 ρ is volumetric density of graphene mass, *a* is thickness of monoatomic layer, k_B is the Boltzmann constant, T is absolute temperature, ε_L is dielectric constant of lattice, e is elementary charge. At sufficiently low temperatures $\theta = \mu_0 = \pi \hbar \Psi_E n_s d$, the

chemical potential of electron gas of graphene with to the linear dispersion law $E_1(k)$:

$$\mu \approx \mu_0 \left[1 - \frac{\theta}{\mu_0} e^{-\mu_0/\theta} \right], \tag{19}$$

n_s is surface concentration of graphene electrons.

Screening of electron-phonon interaction, defined by function $G_{\gamma 1}(\xi)$ (17), is relatively small, since $\mathbf{K}_{F1} = \gamma$ if $\gamma \sim 10^5$ cm⁻¹. Figure 2 shows spectral dependence of the coefficient of absorption $\alpha_1(\gamma)$, according to (13), for graphene ribbon if d = 10Å, $E_a = 5 \text{ eV}$, $n_s = 10^{12} \text{ cm}^{-2}$, T = 4,2K, $\rho = 2.16 \text{ g/cm}^3$, a = 3.34Å, and various values $\xi = q_0 d$. $\alpha_1(\gamma)$ increases with growing depth of potential well $w_0 \propto q_0$ if γ is fixed. $\alpha_1(\gamma)$ passes through its maximum, if γ changes and q_0 remains fixed.

2. Quadratic dispersion law:

$$\begin{split} & E_2(k) = \boldsymbol{h}^2 k^2 u^2 / w_0 \quad \text{provided that} \quad \left| k \right| = q_0, \\ & \cos(2q_0 d) = 0, \ (q_0 c_\gamma / \gamma u) > 1, \ \gamma d = 1. \text{ In this case} \\ & \text{the coefficient of hypersonic absorption will be:} \end{split}$$



Fig. 2. Coefficient of electron absorption of hypersonic of the lowest mode $\alpha_1(\gamma)$ for graphene wire width 20Å and values $\xi : 1 - \pi/8$; $2 - \pi/7$; $3 - \pi/6$; $4 - \pi/5$; $(n_s = 10^{12} \text{ cm}^{-2}, \text{ T} = 4.2 \text{ K})$.

$$\alpha_{2} = \frac{32}{9} \frac{E_{a}^{2} \omega_{\gamma}^{3} \xi(\gamma d)}{h c_{\gamma} u \rho a d B_{\gamma} (c_{d}^{2} k_{1})^{2}} G_{\gamma 2}(\xi) \left[\left(\frac{\xi c_{\gamma}}{\gamma d u} \right)^{2} - 1 \right] [f(E_{\gamma 2}^{(-)}) - f(E_{\gamma 2}^{(+)})] \cdot 8.686 \text{ dB/cm},$$
(20)

$$E_{\gamma 2}^{(\pm)} = \frac{\mathbf{h}\gamma^2 \mathrm{ud}}{4\xi} \left(\frac{\xi c_{\gamma}}{\gamma \mathrm{du}} \pm 1\right)^2, \quad G_{\gamma 2}(\xi) = \left(\frac{\gamma}{\gamma + \mathbf{k} \epsilon_{\mathrm{F2}}}\right)^2, \tag{21}$$

$$\mathbf{\mathring{R}}_{F2} = \frac{8\pi\sqrt{2}e^2n_s}{\varepsilon_L\mu_0} \left(\frac{\xi c_{\gamma}}{\gamma du} - 1\right)^2.$$
(22)

The chemical potential of electron gas with the quadratic dispersion law $E_2(k)$ if $(\theta/\mu_0)^2 = 1$

$$\mu = \mu_0 \left[1 + \frac{\pi^2}{12} \left(\frac{\theta}{\mu_0} \right)^2 \right], \ \mu_0 = \frac{32\mathbf{h}u}{\xi} \pi^2 n_s^2 d^3.$$
(23)

The formula (20) is correct, if the depth of potential well is great enough, when

$$\left(\frac{\xi c_{\gamma}}{\gamma du}\right)^{2} = \left(\frac{q_{0}c_{\gamma}}{\gamma u}\right)^{2} > 1.$$

Evaluation of factor $G_{\gamma 2}(\xi)$ implies that the role of screening of electron-phonon interaction with charge carriers is considerably growing in case of applying the quadratic dispersion law.

Figure 3 a, b, c, d shows dependences between

coefficient of absorption $\alpha_2(\gamma)$ for fixed values $\xi = q_0 d$ and various widths of graphene ribbon. $\alpha_2(\gamma)$ monotonely increases in the zone of considered values γ and is essentially dependent on well depth and ribbon width.

3. Case of small well depth and ribbon width $w_0 \rightarrow 0$ and $d \rightarrow 0$:

$$E_{3}(k) \approx \mathbf{h} | k | u[1 - 2(q_{0}d)^{2}], \qquad c_{\gamma} < \mathbf{h} = 1$$

 $= u[1 - 2(q_0 d)^2], \quad \xi^2 = (q_0 d)^2 = 1, \quad \gamma d = 1.$

The coefficient of hypersonic absorption will be equal to:

$$\alpha_{3}(\gamma) = \frac{8E_{a}^{2}\omega_{\gamma}^{3}\xi^{2}(\gamma d)^{2}}{\mathbf{h}c_{\gamma}\mathcal{U} daB_{\gamma}(c_{d}^{2}k_{1})^{2}} \cdot [1 - (c_{\gamma}/\mathcal{U})^{2}] \cdot G_{\gamma3}(\xi)[f(E_{\gamma3}^{(-)}) - f(E_{\gamma3}^{(+)})] \cdot 8.686 \text{ dB/cm}, \qquad (24)$$

$$\mathbf{E}_{\gamma_{3}}^{(\pm)} = \mathbf{h} \mathcal{C}_{\gamma} \frac{\gamma}{2} \left| 1 \pm \frac{\mathbf{c}_{\gamma}}{\mathcal{C}_{\gamma}} \right|, \ \mathbf{G}_{\gamma_{3}}(\xi) = \left(\frac{\gamma}{\gamma + \mathcal{C}_{F_{3}}} \right)^{2}, \tag{25}$$

$$\mathbf{K}_{F3} = 8 \left(1 - \frac{c_{\gamma}}{\mathbf{k}} \right)^2 \frac{e^2 d}{\varepsilon_L \mathbf{h} \mathbf{k}} (\gamma \xi)^2 .$$
(26)

where



Fig. 3. Coefficient of electron absorption of hypersonic of the lowest mode in graphene wire $\alpha_2(\gamma)$ for the wire width and value ξ : a – 20Å, 1 – 3 $\pi/4$, 2 – 5 $\pi/4$; b – 40Å, 1 – 13 $\pi/4$, 2 – 15 $\pi/4$, 3 – 17 $\pi/4$, 4 – 19 $\pi/4$; c – 80Å, 1 – 85 $\pi/4$, 2 – 87 $\pi/4$, 3 – 89 $\pi/4$; d – 100Å, 1 – 141 $\pi/4$, 2 – 143 $\pi/4$, 3 – 145 $\pi/4$ (n_s = 10¹² cm⁻², T = 4,2 K).

The chemical potential of electron gas, if $(\theta/\mu) = 1$

$$\boldsymbol{\mu} \approx \pi \mathbf{h} \mathcal{U}_{n_s} \mathbf{d} \,. \tag{27}$$

As for case 1 (great depth of potential well w_0), the influence of screening of electron-phonon interaction with charge carriers is quite insignificant.

Fig. 4 shows dependences of $\alpha_3(\gamma)$ for various values ξ in (24). For fixed $\gamma \alpha_3(\gamma)$ increases with ξ growth. If ξ is fixed and d = 10 Å $\alpha_3(\gamma)$ passes through its maximum with γ changing (max $\alpha_3(\gamma)$) is displaced relative to max $\alpha_1(\gamma)$ in the direction of higher values γ).

In general, spectral dependences of the coefficient of intraband electron absorption of hypersound can be rather complicated in straight-line graphene ribbons.



Fig. 4. Coefficient of electron absorption of hypersonic of the lowest mode $\alpha_3(\gamma)$ in graphene wire, for small depth/width of wells and values ξ : 1 - 0, 1, 2 - 0, 12, 3 - 0, 15 ($n_s = 10^{12}$ cm⁻², d = 10Å, T = 4,2 K).

III. Hypersonic absorption with electrons during interband transitions in straight-line graphene ribbons

If $\gamma \mathbf{l}$? 1, where \mathbf{l} is mean free path of electron in the direction of wire axis, the coefficient of hypersonic absorption is expressed by (11):

$$\boldsymbol{\alpha} = \frac{2\pi}{\mathbf{h}c_{\gamma}} \mathbf{g}_{s} \mathbf{g}_{v} \sum_{\mathbf{k}, \mathbf{s}, \mathbf{k}', \mathbf{s}'} \left| \left| \left| \mathbf{\hat{H}} \right| \mathbf{k}, \mathbf{s} \right|^{2} \left(\mathbf{f}_{\mathbf{k}, \mathbf{s}} - \mathbf{f}_{\mathbf{k}', \mathbf{s}'} \right) \cdot \boldsymbol{\delta} (\mathbf{E}_{\mathbf{k}', \mathbf{s}'} - \mathbf{E}_{\mathbf{k}, \mathbf{s}} - \mathbf{h}\omega_{\gamma}) \quad ,$$
(27)

Below we summarize the final results for coefficient α of electron absorption of hypersound in straight-line graphene ribbon during interband transition

 $s = -1 \rightarrow s' = 1$. When

$$E_{k,1}(k) = \mathbf{h} |k| \mathscr{H}_{F}, |k| = q_{0}, \cos(2q_{0}d) \neq 0, c_{\gamma} > \mathscr{H}_{F} = u |\cos(2q_{0}d)|, \ \gamma d = 1$$
(28)

we will have:

$$\alpha = \frac{E_a^2 G_{\gamma l}(\xi)(\gamma d)^2 \omega_{\gamma}^3}{2\mathbf{h} c_{\gamma} u \rho a d B_{\gamma} (c_d^2 k_l)^2} \left(\frac{c_{\gamma}^2}{\mathscr{V}_F^2} - 1 \right) F_l(\xi) [f(-\mathbf{h} \mathscr{V}_F \gamma^{(-)}) - f(\mathbf{h} \mathscr{V}_F \gamma^{(+)})] \cdot 8.686 \text{ dB/cm},$$
(29)

$$F_{1}(\xi) = \frac{(\sin^{4} 2\xi)(1 - \varphi_{1}^{(+)}\varphi_{1}^{(-)})^{2}}{\xi^{2} |\cos(2\xi)| (\sin^{2} \xi + \varphi^{(+)2})(\sin^{2} \xi + \varphi^{(-)2})},$$
(30)

$$\varphi_{1}^{(\pm)} = \frac{|\cos(2\xi)|\cos\xi \pm (|\sin(2\xi)| \pm 1)\sin\xi}{|\cos(2\xi)|\sin\xi \mathbf{m}(|\sin(2\xi)| \pm 1)\sin\xi},$$
(31)

The expression \mathbf{B}_{γ} in (29) is calculated by formulae (II1)-(II3) in [12]; $\mathbf{K}_{\gamma 1}$ is calculated by formulae (18) and (16) with taking into account (14), (17); ρ is volumetric density of graphene mass, *a* is thickness of monoatomic layer, $\mathbf{k}_{\rm B}$ is the Boltzmann constant, T is absolute temperature. The chemical potential of electron

gas of graphene is as follows: $\mu_0 = \pi h \mathscr{K}_F n_s d$, applying the linear dispersion law $E_{k,1}$ at sufficiently low temperatures, therein n_s is surface concentration of graphene electrons. It is possible to ignore screening of electron-phonon interaction, if $(10^5 - 10^6) \text{ cm}^{-1}$ for the case of linear dispersion law of charge carriers.



Fig. 5. Coefficient of interband absorption of hypersound of the lowest mode in graphene wire $\alpha(\gamma)$, if $\cos(2\xi)$: 1 - 0.002, 2 - 0.003, 3 - 0.004 and wire half-width: a - 10Å, b - 20Å (n_s = 10¹³ cm⁻², T = 4.2K).

It should be noted that the interband mechanism of hypersonic absorption, according to energy-impulse conservation laws, is only if $c_{\gamma} > \frac{1}{2}$, while the intraband mechanism is if $c_{\gamma} < \frac{1}{2}$. As $(c_{\gamma}/u) = 1$ permanently, the following condition shall be fulfilled for interband mechanism activation according to linear dispersion law of electrons:

$$0 < |\cos(2q_0 d)| < (c_{\gamma} / u).$$
 (32)

In case of small depth of well w_0 and width 2d of graphene ribbon, when $\xi = q_0 d \rightarrow 0$, the interband absorption of hypersound is absolutely impossible, in contrast to the intraband mechanism.

In another limiting case [5], if $|\mathbf{k}| = q_0$, $\gamma d = 1$ and accurate equality

$$ps(2q_0d) = 0$$
, (33)

when the quadratic dispersion law is observed for electrons (with energy gap W_0)

$$E_{k,1} = \frac{\mathbf{h}^2 k^2 u^2}{w_0}, \ E_{k,-1} = -w_0 - \frac{\mathbf{h}^2 k^2 u^2}{w_0}, \qquad (34)$$

it is possible to ignore interband absorption within the accepted approximation, because the result of applying conservation laws in case of acoustic phonon absorption contradicts the condition of (33), while intraband absorption is substantial for the given case.

Figure 5 shows the spectral dependence of the coefficient of interband absorption $\alpha(\gamma)$ by (28),(29) and taking into account (32) for graphene ribbon, if d = 10Å, 20Å, $E_a = 5 \text{ eV}$, $n_s = 10^{13} \text{ cm}^{-2}$, T = 4.2 K, $\rho = 2.16 \text{ g/cm}^3$, a = 3.34 Å, and various values $\xi = q_0 d$ under the condition of (32). The coefficient of interband absorption of hypersound is monotonely growing from γ for the considered actual values of γ . The value $\alpha(\gamma)$ reaches its maximum in extrapolation of formula (29) in the zone $\gamma d \gtrsim 1$, and reduces afterwards. It should be also noted that interband absorption reduces, when d grows for fixed equal values $\cos 2\xi = \cos 2q_0 d$, if $q_0 = w_0 / hu$ decreases, i.e. the depth of potential well w₀ of graphene wire. As concerns the constant width of wire 2d, the interband absorption diminishes with increasing depth of potential well (Fig. 5). Fig. 6 shows the dependence $\alpha_{_0}(\gamma)$ if T = 0 and $\mu = 0$, when the Fermi level is located in the Dirac point [1,2]. In this case, interband absorption of hypersound prevails over intraband one.

Conclusions

A straight-line graphene wire is a waveguide with specific system of hybride acoustic modes of confined phonons. At low temperatures, the main mechanism of



Fig. 6. Coefficient of interband absorption of hypersound of the lowest mode in graphene wire $\alpha_0(\gamma)$, if $\cos(2\xi) = 0.004$ and wire half-width: a - 10Å, b - 20Å, c - 40Å, d - 50Å ($n_s=0$, $\mu=0$, T=0).

hypersound absorption is electronic mechanism at intraband transitions in the model of straight-line graphene ribbon. The obtained spectral dependences of the coefficient of hypersound absorption are considerably determined by the dispersion laws of Dirac bound electron states, which are significantly dependent on potential well depth and graphene ribbon width. The screening of electron-phonon interaction with charge carriers is insignificant for high-frequency sound in the considered model, when applying the linear dispersion laws. We considered interband absorption of hypersound for the limiting laws of dispersion of Dirac's electron states. It shows that appearance of interband mechanism is substantial for a certain limiting case of linear dispersion law under quite stringent conditions, connected with potential well depth and The graphene wire width. obtained spectral dependences of interband absorption of hypersound demonstrate an important role of quantum-size limits both for electrons and phonons of a wire. One should take into account not only intraband mechanism of hypersound absorption in the model of straight-line graphene ribbon at low temperatures, but also interband mechanism, the experimental research of which enables, in particular, to define depth of potential well of graphene wire.

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- [1] A.K.Geim and K.S.Novoselov, Nature materials, 6, 183 (2007).
- [2] K.S.Novoselov et al, Nature, 438, 197 (2005).
- [3] L.Brey and H.A.Fertig, arXiv: cond-mat/0603107 1, 1 (2006).
- [4] F. Sols, F. Guinea and A.H. Castro Neto, Phys.Rev.Lett. 99, 166803-1 (2007).
- [5] T.Ja. Tudorovskij, A.V. Chaplik, Pis'ma v ZhJeTF, 84 (11), 735 (2006).
- [6] V.L.Gurevich. Kinetika fononnyh sistem, Nauka, M. (1980).
- [7] S.G. Alekseev, Ju.V. Guljaev, I.M. Koteljanskij, G.D. Mansfel'd. UFN, 175(8), 895 (2005).
- [8] T. Hesjedal. Proc. of SPIE, 5045, 11 (2003).
- [9] L.Wirtz and A.Rubio, arXiv: cond-mat/0404637 1, 1 (2004).
- [10] M.A. Ruvins'kij, B.M. Ruvins'kij, Fizika i himija tverdogo tila, 9(2), 309 (2008).
- [11] V.F. Gantmaher, I.V. Levinson, Rassejanie nositelej toka v metallah i poluprovodnikah. Nauka, M. (1984).
- [12] M.A. Ruvinskij, B.M. Ruvinskij, FTT, 49(11), 2076 (2007).

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Про поглинання гіперзвуку в прямолінійних смужках графену

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Досліджено внутрішньозонний та міжзонний механізми електронного поглинання гіперзвуку. Визначено коефіцієнти поглинання обмежених акустичних фононів гіперзвуку з урахуванням екранування деформаційної електрон-фононної взаємодії носіями заряду та законів дисперсії діраківськмх електронних станів.